Teaching-Learning Plan for the Research Lesson

**Elementary School**

**Grade Level: First**

**1. Title of Lesson: Mathematical Thinking and Problem Solving in First Grade**

**2. Goals: (Main Aim, Broad Subject-Matter Goals, Lesson Goals)**

*“It is important to define goals hierarchically because children’s knowing how to compute is worthless in the long run if they do not do their own thinking and have no confidence, no number sense, and no ability to exchange ideas with other people. If, on the other hand, children do their own thinking and interact well with others, they will all eventually become able to solve the kinds of problems found in arithmetic.”*

*- Constance Kamii*

Algebraic problem solving is a tricky section on the new district assessment. Our inquiry will look at what kinds of problem situations we should be introducing at First Grade.

We also want to learn more about teacher moves which invite students to construct understanding of the meaning of the equal sign (relational thinking).

In Algebraic Thinking, students in First Grade will come to recognize how a number sentence can represent the problem situation. This understanding will be developed through manipulatives, pictorial representation, oral communication, and mathematical symbols.

For this lesson, given different problem structures, students will represent the problem with self-selected tools and strategies, justify their thinking, and use the shared knowledge of others to strengthen their own explanation and critiquing skills.

**3. Lesson Rationale: Why we chose to focus on this topic and goals. (For example, what is difficult about learning/teaching this topic? What do we notice about students currently as learners?) Why we designed the lesson as shown below.**

*“Elementary mathematics is not superficial at all, and anyone who teaches it has to study hard in order to understand it in a comprehensive way.” - Liping Ma (p. 146)*

Growth Mindset:

We chose this lesson because we wanted to see how can we foster a growth mindset that will help students take risks in problem solving with a group, in partners, and independently. We were interested in what this mindset would look like and sound like as the difficulty of problem situations changed. Developing confidence and persistence was a focal area.

Making Thinking Visible:

We also wanted to make thinking visible, focusing on student talk and teacher talk as the driving force. Throughout the lesson design process, we honed in on effective guiding questions that encouraged mathematical reasoning. We refined questions based on student responses, both oral and written, paying attention to the needs of a range of learners (See “Ahas and Insights”). Constance Kamii described this learning process well, “Children’s first methods are admittedly inefficient. However, if they are free to do their own thinking, they invent increasingly efficient procedures just as our ancestors did. By trying to bypass the constructive process, we prevent them from making sense of arithmetic. (p. 32)”

Using Data to Guide Instruction:

We looked at our mid-year assessment to analyze where students were having difficulty. We determined that equality equations were abstract and a point of difficulty. We decided to add scaffolds by presenting equality situations within a word problem context.

Choosing Problem Structures:

We looked at the California Math Frameworks, specifically at the problem examples for first grade. We decided to look at Start Unknown and Change Unknown situations because these were the structures that were most challenging for students to interpret.

As we looked at student work, the following questions guided our inquiry:

● How are students constructing number models given different problem structures?

● Are they able to maneuver the numbers around the equals sign?

Modifying the Lesson (Three times):

In the first dirty lesson, we presented 3 problems for the students to solve. Before sending them off to work, we brainstormed a list of ways to show one’s thinking when solving problems. The students worked independently, solving the problems using self-chosen strategies and tools. We pulled the students back together as a whole group between each problem. We read each problem to the students before they went off to work. Upon completion of the final problem, the students went on a gallery walk to observe how others had solved the problem and shown their thinking. We found that the students were tired by the third problem. They were no longer thinking through the problems carefully. We also felt that we had left out an important part of the lesson, discussing our work. The students never had the opportunity to share their thinking with a partner or the whole group.

The second lesson was designed with embedded partner work, so the group could observe how this might impact student results. Based on the first lesson, we reduced the number of problems presented, and we included a math talk between the two problems to scaffold the thinking of students so that they might apply the learning to the next problem. Finally, the teacher presented several specifically selected examples of student work that might again challenge student thinking, inviting the class to compare the work and justify their thinking.

The third lesson built on the first two lessons and incorporated our new learnings. In the opening math talk, students recalled the tools and strategies that had been used in their previous work and together determined tools and strategies used on another student example. The class discussed how to solve a change unknown problem on the rug, then went to their seats to solve and record their thinking for a new problem. Students worked independently on this first problem, then began a new problem involving start unknown. Since we wanted to encourage partner talk and comparison of student work, we introduced several questions.

**4. How does students’ understanding of this topic develop? How does it fit within students’ experiences in prior and subsequent grades?**

*“Young children are able to solve a variety of story problems using strategies that range from modeling the problem with counters (including fingers) to recalling basic facts or using derived facts.”- Carpenter, Carey, and Kouba (1990)*

Many students (and even some adults!) think that the equal sign means “the answer comes next.” (RAND Mathematics Study Panel, 2003). Van de Walle teaches us that “the equal sign is rarely represented in US textbooks in ways that facilitate children’s understanding of the equivalence relationship - an understanding that is critical to understanding algebra.” (McNeil et al., 2006) (Teaching Student-Centered Mathematics p. 230)

Students have been working on problem solving and showing their thinking when solving the problem. Students have been looking collectively with help of the teacher at examples of student work and talking about what they see in the work that is helpful for solving the problem, and evidence of what tools the student worked with to find a solution. From John VandeWalle, we learned, “Having children share their reasoning promotes relational thinking and can help other children to improve analyzing relationships in a problem.” (Teaching Student-Centered Mathematics p. 236)

Students have had experiences in using addition and subtraction interchangeably. According to Van de Walle, “Too often our story problems have result unknown. But children need many experiences with other parts missing….” That is why we wanted to give students an opportunity to solve problems with *start unknown and change unknown.*

Evolution of children’s strategies (Carpenter, p. 5):

1. Direct modeling of action and relations in problem

2. Counting strategies

3. Number facts/properties of operation and number sense

4. Base ten concepts

**5. Relationship of the lesson to Common Core Standards**

**Represent and solve problems involving addition and subtraction.**

**1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

**1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

**Understand and apply properties of operations and the relationship between addition and subtraction.**

**1.OA.3**. Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so

2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

**1.OA.4** Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

**Math Practice Standards**

1 - Make sense of problems and persevere in solving them.

2 - Reason abstractly and quantitatively.

4 - Model with mathematics.

5 - Use appropriate tools strategically.

6 - Attend to precision.

7 - Look for and make use of structure.

**6. Lesson Design:**

Research questions:

● How are they solving it?

● Who is struggling? What would you do for someone who is stumped? Who needs more of a challenge? (Do we need to have the same numbers for everyone? Shall we observe in Lesson 1 and then change for Lesson 2?)

● What guiding questions are effective to spark new thinking?

● Categorize the student strategies based on our observations? (Counting up/down…) What progression of strategies do we need to know in order to respond effectively within the lesson?

● What progression of problems makes the most sense?

|  |  |  |  |
| --- | --- | --- | --- |
| **Student Learning Activities** | **Anticipated Student Responses** | **Points to Notice (Evaluation)** | **Materials/Prep** |
| **Math Talk Warm Up**  What are some ways to show your thinking?    **Teacher Prompts:**  Suggest the use of manipulatives, as appropriate.  I have some interesting problems for you to solve.    **1st Problem with Partners (Change Unknown) 10 minutes:**  Colleen had 15 pencils. She gave some pencils to Roger. Then she had **8** pencils left. How many pencils did Colleen give Roger?    Visualize - What happened in the problem?  What do you know?  What do you need to find out?  \*Think Time\*  Share with your partner  Complete worksheet with partners with different colored pens.    Share out and record strategies (go back to original list made at math talk)    **2nd Problem at desks (Start Unknown):**  Colleen had some pencils. She gave 7 pencils to Roger. She has 8 pencils left. How many did she start with?    What model can you make?  Point to tools around the tools you can use?  Did you know that fingers are your best tools?    **Sharing:**  Talk with your partner and share the strategy that you used.  Did your partner try something that was the same? Different?    Come back to the rug and review more to first question about “Show your thinking”    **Closure:**  \*Teacher selects 3 different strategy examples.  What are you noticing that’s the same in people’s strategies? What’s different? | **Math Talk Warm Up**  \*Manipulatives (linker cubes, counters)  \*Drawing  \*Fingers  \*Number Grid  \*Number Line      **1st Problem:**  15 - \_\_\_ = 8    **Sample Responses:**  Number sentences  15 - 8 = 7  8 + 7 = 15  Pictures  Words                                            **2nd Problem:**  7+8 = 15  15-7 = 8  15-8 = 7  \_\_\_-7 = 8                    **Sharing:**  Shared strategies from earlier list | **Listen**                      **Observe for strategy(ies) used.**      **Observe for perseverance through problem.**      **Observe same group of students and their thinking.** | -Chart paper    -Linker cubes    -Enlarged worksheet 1st problem (change unknown)    -Worksheet for 2nd problem (Start unknown)    -2 different colored pens    -Questions on chart paper |

**7. Data collection points during the lesson observation.**

Our team will collect data on:

● **Are students able to identify the “start unknown”?**

● **Are they representing their thinking in words, numbers and pictures?**

● **How are students constructing Number Models?**

● **Using the equal sign ? What does it mean?**

● **Collect student work.**

Outside observers are asked to collect data on:

● **Same as above**

**Conclusion: What we have learned from this lesson study process:**

*“ Making sense is at the heart of mathematics and so it must also be at the heart of the mathematics we do with young children”- Kathy Richardson*

Numbers on paper are an abstract representation. For young children we are helping them move from understanding concretely to understanding abstractly. We are excited to be learning about how teacher language, student conversation centered on student work, manipulatives, and problem situations can help foster that understanding.

**Ahas and Insights**

*“Because of intimate knowledge you have of your students, you are the most qualified to make immediate decisions about supporting your own students.” -Thomas Carpenter*

Lesson Study has been a powerful professional development model that allows us to look closely at big ideas in mathematics. Not only do we learn pedagogical strategies to meet the needs of all the students within our class, we are able to simultaneously learn math concept knowledge deeply, which is required for us to make thoughtful and impactful minute-by-minute instructional decisions that guide our students to ever-increasing levels of understanding.

Liping Ma talks about “knowledge packages” that effective teachers of mathematics construct. When studying Chinese educators, she found that these teachers developed deep ideas on how math concepts (procedural knowledge, conceptual knowledge, basic principles) are related and structured. In her words,

“A fully developed and well-organized knowledge package about a topic is a result of deliberate study (p. 22).” Lesson Study provides the forum for deliberate study. As you read our Ahas and Insights, you will find our research led to additional questions that helped us redesign our lesson. Each lesson observation helped us learn about how ideas in math are connected. This personal growth on the part of the teacher helped us respond to student learning needs by knowing which question to pose to move student thinking forward.

What follows is a list of categories that summarizes many of the mathematical and pedagogical ideas that developed as a result of our discussions, lesson designs, and analysis of qualitative and quantitative data on learning and teaching. Ahas and insights are bulleted.

Turn and Talk and Guiding Questions

One of the most interesting areas of inquiry revolved around the power of talk in the math workshop--both student and teacher talk. We explored structures to increase student talk within the lesson as well as effective guiding questions that would prompt mathematical thinking.

● In lesson 2 and 3 we deliberately added more opportunities for Turn and Talk. When we did this, we saw an increased engagement in the lesson. Students were also more connected to the problem as they were able to talk about the numbers in the problem and what the numbers actually meant. For example, in the problem *“Colleen had 15 pencils. She gave some pencils to Roger. Then she had* ***8*** *pencils left. How many pencils did Colleen give Roger?”*  some students discussed the quantity of pencils Colleen started with 15 pencils. Students noticed that 15 was a large number. By giving pencils away, students knew they were decreasing the number of pencils. Some students were able to use this understanding to help solve the problem, others knew they had to subtract, but did not know how because the structure of the problem was “Change Unknown,” a challenging and unfamiliar structure for many students.

● Visit<http://old.newteachercenter.org/collaborative-discussions/turn-and-talk/plan> for more information on the Turn and Talk strategy.

● Prompting student thinking was explored through the use of guiding questions during whole group instruction and independent/paired problem solving. We created a list of the questions we found to be most effective in illuminating student thinking while encouraging agency. Questions that were more open-ended allowed for students to describe strategies matching their level of understanding, like “How do you know?” When children struggled, specific questions could help them approach the problem using what they knew in combination with information given in the story problem. (Would it help to draw a picture?) Exploring questions that help students rephrase other student thinking, agree or disagree with an idea, and elaborate on ideas are areas for further study.

Guiding Questions That Prompt Mathematical Thinking and Reasoning

Getting Started With a Problem:

● What are you trying to find out?

● Would it help to draw a picture to know what to do? (help visualize the problem)

● Would you get the same answer if you…?

Windows Into Student Thinking

● Tell me what you’ve done.

● What did you do next?

● What are you going to do next?

● How did you get that?

● Can you draw a picture to represent this?

Encouraging self-correction/Addressing Misconceptions

● I don’t understand. Walk me through this.

● I’m not sure about that. This is the part I’m not sure about…

● You said \_\_\_\_\_\_, so \_\_\_\_\_\_\_\_. (encourage conflict in thinking)

Revisiting Problems

● Any way you work the problem is okay, but I want you to think about the fastest and easiest way to \_\_\_\_\_\_\_

● Does anyone notice anything about \_\_\_\_\_\_\_\_\_’s strategy?

● What is similar/different about these strategies?

● Do you think that always works?

● Why does that work?

Varied Structures of Word Problems

One of our initial questions that surfaced before the first dirty lesson involved different structures of word problems. We felt confident that our students would solve “Result Unknown” problems because they had been exposed to this structure within addition and subtraction frequently. “Change Unknown” and “Start Unknown” were less familiar. We had no idea how the change in structure would affect the students.

We also wondered whether to give the class one problem to solve or to change the numbers for some, increasing the numbers for a challenge or decreasing the numbers to simplify. In the end, we agreed that having the class solve the same problem built community and allowed for a rich discussion with shared viewpoints and varied strategies. Students who finished quickly were asked to represent their thinking in a different way or were asked to write a similar problem with a different context on the back for a partner to solve. Students who struggled were able to listen to the ideas of peers.

For the first lesson, we wanted to collect data to get a clearer picture of how our young mathematicians would respond to the challenge.

● Because we collected written responses, we were able to get a window into student mathematical thinking by looking at how they represented their problem solving process using pictures, numbers, and words. In addition, an adult was assigned a table to transcribe student behavior and oral verbalizations.

● After the first lesson, we realized that many students reached for counters immediately to model the problem. As they solved more challenging problems, strategies became more varied. Some students could solve the problem using equations, but they could not explain what the numbers meant or why they chose to put the numbers and symbols in the order within the equation.

● Some students, when asked “How did you get that?” put related facts to justify their answer. They listed known fact families (8+7=15; 7+8=15; 15-8=7; 15-7=8). They could not explain how the number sentences connected to the problem. As a result of this response, in the revised lesson #2, we added the recording of number sentences by the teacher during whole class discussions to represent student thinking. We believe this step is critical to help kids connect the numbers in the story to the action in the story. It is the necessary scaffold to help kids bridge from the conceptual to the abstract. Liping Ma states, “In discussions, students may report, display, explain and argue for their own solutions. Through the discussions, the explicit construction of links between understood actions on the objects and related symbol procedures would be established.” Ma’s understanding of how students transition from conceptual modeling to the symbolic mirrors our research findings.

● A crucial part of the lesson was to circulate during the problem solving times (independent and partnered). Circulating allowed us to work individually or in pairs with students, differentiating instruction for a range of learners. Instead of telling the students answers or leading them to a predetermined algorithm, we found that using guiding questions led to deeper thinking, increased agency, and an increase in persistence to stick with the problem and try multiple approaches. This quote from Constance Kamii helped us solidify our thinking on independence, “To encourage children to do their own thinking, the teacher refrains from reinforcing right answers and correcting wrong ones, instead encouraging children to agree and disagree with one another.” Marilyn Burns also talks about the importance of students’ justifying their thinking: “It’s important to remember that students’ correct answers, without accompanying explanations of how they reason, are not sufficient for judging mathematical understanding.” We would like to continue to research accountable talk strategies to help kids justify their thinking and critique the thinking of others, thoughtfully.

Kids Inventing Their Own Strategies/ Teachers Valuing a Variety of Approaches

Throughout the lessons, we kept debating the need to help the children make connections to the story structure and the number sentence they recorded. We encouraged counters and pictorial representations. However, we were also asking the students to write a number sentence. What happens if the number sentence they wrote did not match the structure of the story problem? Was using addition to solve a problem that required subtraction an incorrect strategy?

To illustrate, consider this problem: *Colleen had some pencils. She gave 7 pencils to Roger. She has 8 pencils left. How many did she start with?* The equation that matches the structure would look like this: ? - 7 = 8

As we watched students wrestle with this problem, we discovered most children recorded an addition number sentence (7+8=15). When asked to explain why they used this strategy, only a few students could justify that they had to add the difference to the subtrahend to get the minuend. Were we expecting too much? What should we expect as a first grade response to this problem?

We researched this and discovered:

● Very few children naturally use subtraction. They use addition strategies, and if they use subtraction they quickly change to other strategies.- Kamii (p. 74)

● One of the part-whole relationships in subtraction, even with single-digit numbers, is much more complicated than in addition. In addition such as 5+4, the child begins with 2 wholes, 5 and 4, at the same hierarchical level and combines them into a higher-order whole in which the previous wholes become parts (arrow in ONE direction).

|  |  |
| --- | --- |
|  |  |
|  |  |

On the other hand, in subtraction, such as 9-5, the child has to deal

|  |  |
| --- | --- |
|  |  |
|  |  |

simultaneously with the whole, 9, and the part, 5, which are two hierarchical levels

(arrow in TWO directions).

Addition only involves “ascending” from the parts to the whole in one direction.

Subtraction, on the other hand, entails both “ascending” (from parts to the whole)

and “descending” (from the whole to the parts). This thinking in two opposite

directions simultaneously is so difficult...- Kamii (pp. 75-76)

Kamii also stated on p. 67, “We let computational procedures come out of word problems and do not say what operation must be used. The reason for our approach is that, in the elementary grades, logico-mathematical knowledge is still growing out of children’s mental actions on concrete objects.” We took this to mean that we should be encouraging and expecting a range of representations based on students’ levels of mathematical understanding.

Asking our students to write ? -8= 7 does not match THEIR thinking, even if it matches the structure of the problem. Most students understood that they had to figure out the start and could do so with addition of the difference and subtrahend. They could explain that Colleen had started with a large number and that the action in the problem was “giving away” a quantity, reducing the number. They used counters and number sentences to explain the process of adding to arrive at the start unknown. However, writing the second number sentence to match the structure of the story was beyond most students at this age level.

Another strategy that came up quite frequently was the use of fingers as a tool for counting, counting being the strategy a child would use to solve addition or subtraction problems. We encouraged the use of fingers as a math tool based on research. Thomas Carpenter (p. 29) states, “Although children frequently use fingers with Counting strategies, the use of fingers does not distinguish Counting strategies from Direct Modeling strategies….Fingers may be used to directly model a problem or to keep track of the steps in a counting sequence.” We also looked at research and activities on YouCubed.org by Dr. Jo Boaler.

We found examples of students using fingers as a counting strategy. Many students numbered the fingers in a forward or backward sequence, depending on the problem and whether they were counting forward or backward. Fingers became a number line of sorts. These students almost always recorded a number sentence below the “finger number line.” Other students used fingers as a direct modeling strategy. These students drew hands with 5 fingers. One child drew 3 hands, with 5 fingers each (unlabeled) to represent the quantity 15. The variety of strategies were fascinating and provided a window into student thinking.

Relational Thinking in First Grade:

Relational thinking kept reappearing in our conversations throughout this lesson study. It started with our analysis of the district math assessment. Many students were confused with how to explain the equal sign in words and within equations, especially when the equations were not familiar (ex. True or False: 9 = 8+1)

We found that many students thought that the equal sign meant “the answer comes next” or “is.” This proved problematic when they were given unfamiliar number sentences like 10= ? +6 or 8+2= 5 + 5). Using the metaphor of a balance scale to help students visualize the equal sign as “the same as” is a strategy many of the teachers are now using to help students understand how the symbol is used to represent relationships between numbers.

Being able to understand and represent ideas symbolically using numbers and symbols, like the equal sign, gives students a powerful way to express their thinking. It helps students record their ideas and reread it, allowing them to discuss and compare strategies, transforming their conceptual understanding into a symbolic, generalizable notation. This is the beginning of algebraic reasoning in first grade.

**Questions we have now:**

● How can we leverage purposeful partnerships within discussions using the Turn and Talk model to create opportunities for students to be influenced by the thinking of others?

● How can we leverage purposeful partnerships within the problem solving portion of the lesson? Is it best to let the students work on the problem independently at first and then partner to share thinking?

● Does order of presentation matter when presenting word problems of different structures? Should result unknown be used first? Which combination of problems will move students’ thinking forward most effectively?

● Changing the numbers can alter accessibility to the problem. Which number combinations are the best to start with? Which combinations will provide the right challenge so students can focus on strategies and justification of approaches?

● What guiding questions can lead to more accountable talk (students holding onto a line of thinking, elaborating on an idea, arguing respectfully for the most effective strategy…)?

● When should we introduce variables into number sentences?

**Bibliography**

Burns, Marilyn. (2007). “Nine Ways to Catch Kids Up”. *Educational Leadership,*

November 2007, pp. 16-21.

Carpenter, Thomas P., et al. (2015). *Children’s Mathematics: Cognitively Guided*

*Instruction.*Portsmouth, NH: Heinemann.

Kamii, Constance. (1994). *Young Children Continue to Reinvent Arithmetic.* New York:

Teachers College Press.

Ma, Liping. (1999). *Knowing and Teaching Elementary Mathematics.* Mahwah, New

Jersey: Lawrence Erlbaum Associates, Publishers.

Van de Walle et al. (2014) *Teaching Student-Centered Mathematics: Developmentally*

*Appropriate Strategies for Grades Pre-K-2, Volume 1 (2nd Ed.)*. Boston, MA:

Pearson.