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Teacher Questioning to Elicit Students' Mathematical Thinking in Elementary School Classrooms

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Cognitively Guided Instruction (CGI) researchers have found that while teachers readily ask initial questions to elicit students' mathematical thinking, they struggle with how to follow up on student ideas. This study examines the classrooms of three teachers who had engaged in algebraic reasoning CGI professional development. We detail teachers' questions and how they relate to students' making explicit their complete and correct explanations. We found that after the initial "How did you get that?" question, a great deal of variability existed among teachers' questions and students' responses.

Keywords: *teaching; mathematics; elementary*

Across the curriculum teachers are being asked to delve into and make use of students' thinking. Mathematics is no exception. Mathematics education researchers have gathered consistent evidence of the benefits of attending to students' thinking (Franke, Kazemi, & Battey, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sfard & Kieran, 2001; Silver & Stein, 1996). During the past 20 years, researchers investigating Cognitively Guided Instruction have worked with teachers, sharing research about the development of students' mathematical thinking and studying teachers' use of that information. These researchers have found that teachers readily begin asking students open-ended questions after the students have solved a problem (e.g., "How did you solve that problem?") and can elicit an initial student explanation. Teachers find it more difficult, however, to follow up on student explanations and pursue students' thinking in ways that support students as they try to detail their strategies or connect with other students' strategies (Franke, Fennema, Carpenter, Ansell, & Behrend, 1998). Little

research-based evidence exists to help teachers make the transition from asking the initial question to pursuing student thinking. We know little about the details of teacher practice, specifically the kinds of questions a teacher asks when supporting students in making their thinking explicit.

Background

Student talk is considered a major component of classroom discourse and a vehicle for increasing student learning:

Students must talk, with one another as well as in response to the teacher. . . . When students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively,

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revealing mathematics as constructed by human beings within an intellectual community. (National Council of Teachers of Mathematics, 1991, p. 34)

Student talk can lead to increased student mathematical knowledge and understanding in two interrelated ways. First, listening to students talk makes it possible for the teacher (and other students) to monitor students' mathematical thinking. Teachers can use information gleaned from student talk to inform their instructional decision-making practices, including problems to pose and follow-up questions to ask (Franke, Fennema, & Carpenter, 1997). Similarly, when students converse with each other, their talk makes it possible for students to gauge each other's strategies and comprehension, providing opportunities for students to help each other build more complete mathematical understanding. Second, the act of talking can itself help students develop improved understanding. Describing, explaining, and justifying one's thinking all help students internalize principles, construct specific inference rules for solving problems, become aware of misunderstandings and lack of understanding (Chi, 2000), reorganize and clarify material in their own minds, fill in gaps in understanding, internalize and acquire new strategies and knowledge, and develop new perspectives and understanding (Bargh & Schul, 1980; King, 1992; Rogoff, 1991).

Not just any kind of student talk is expected to be productive for supporting or challenging students' thinking. Providing explanations is positively related to achievement outcomes, even when prior achievement is controlled for, whereas giving only answers is not related or is negatively related to achievement outcomes (Webb & Palincsar, 1996). Beyond providing answers, students must describe how they solve problems and why they propose certain strategies and approaches. Moreover, when describing their thinking, students must be precise and explicit in their talk, especially providing enough detail and making referents clear so that the teacher and fellow classmates can understand their ideas (Nathan & Knuth, 2003; Sfard & Kieran, 2001).

Teacher Moves to Support Student Explanations

Despite the demonstrated importance of students' explaining their thinking, "teacher-centered instruction continues to dominate elementary and secondary classrooms" (Cuban, 1993). In most classrooms students infrequently ask questions (Graesser & Person, 1994), and teacher talk typically dominates classroom discourse (Cazden, 2001). Moreover, the vast majority of teacher queries consist of short-answer, low-level questions that require students to recall facts, rules, and procedures (Graesser & Person, 1994),

rather than high-level questions that require students to draw inferences and synthesize ideas (Hiebert & Wearne, 1993; Webb, Nemer, & Ing, 2006). International comparisons mirror these findings. The lack of opportunities in U.S. classrooms for students to discuss connections among mathematical ideas and to reason about mathematical concepts constituted one of the most prominent findings of the Third International Mathematics and Science Study (Hiebert et al., 2003; Stigler & Hiebert, 1999). Additionally, these descriptions of the level of student participation echo those made two or more decades ago (e.g., Cazden, 1986; Doyle, 1985; Gall, 1984; Mehan, 1985).

Yet we know from a growing body of work that teachers' questions scaffold students' engagement with the task, shape the nature of the classroom environment, and create opportunities for learning high-level mathematics (Boaler & Brodie, 2004; Kazemi & Stipek, 2001; Smith, 2000; Stein, Remillard, & Smith, 2007). We also know that teachers' questions can serve as a way to move students through the task in a specified way, ensuring they get the correct answer (Wood, Cobb, & Yackel, 1991). Finding the balance in the types of questions and when to ask them can make a large difference in how students continue to participate.

A number of researchers have begun to make explicit the moves a teacher may make to support students in making their mathematical thinking explicit, such as asking students to share their ideas publicly and using those ideas as the basis of conversation. For example, Wood (1998) examined the role of the teacher when supporting students to make explanations and found that teachers used different approaches: taking on some of the mathematical work and moving students in a direction teachers thought most critical ("funneling") versus encouraging students to do most of the mathematical work by focusing attention on particular aspects of students' explanations without guiding students in a specific, predetermined direction ("focusing").

Much remains to be learned about how teacher questioning in mathematics classrooms can help students participate in ways that allow them to make explicit their mathematical thinking and lead them to formulate complete and correct strategies. In this study, we look closely at the questions teachers ask as they engage with their students in mathematical conversation and the ways in which students participate in relation to teacher questioning.

Method

Building on the large-scale study of professional development by Jacobs and colleagues (2007), we selected teachers who had been engaged in the algebraic reasoning professional development for more than a year. In this article, we focus on three classrooms and the ways the teachers asked questions to help students make

public and extend their mathematical thinking. We chose these teachers for observation and analysis because they came from similar schools, taught similar concepts and skills, used similar classroom structures (a combination of collaborative group and whole-class discussion of problem-solving strategies), but showed substantial differences in student achievement on posttests of algebraic thinking. We videotaped and audiotaped conversations in these classrooms in ways that allowed us to document what students said to the teacher and to each other so that we could closely analyze the relationship between teacher practice and student participation.

Participants

The three elementary school classrooms (two second grade, one third grade) analyzed here come from a large urban school district in Southern California. Prior to the algebraic thinking professional development and the large-scale study, the district administrators and teachers recognized the value of engaging in algebraic reasoning in elementary school, and long-term plans for overall school improvement were under way. The district, in its 2nd year of new leadership when the study began, had a history of poor performance and a long-standing sense from those outside the district that it would never do well. According to the state's ranking system and standardized test scores, it was one of the lowest performing school districts in California. As in many urban school districts, hiring and retaining qualified teachers was a struggle. Although the district was making progress, at the beginning of the study, only 57% of the teachers in the district held credentials and 30% of the teachers were in their 1st or 2nd year of teaching. The community served by this district had shifted from being predominantly African American to being predominantly Latino, and at the time of our work, the schools served students of whom 99% were minority, 52% were classified as English language learners, and 93% received free or reduced-cost lunch.

Professional Development Program

Participating teachers engaged in professional development to explore the development of students' algebraic reasoning and, in particular, how that reasoning could support students' understanding of arithmetic. The professional development content, drawn from *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School* (Carpenter, Franke, & Levi, 2003), highlighted "relational thinking," including (a) understanding the equal sign as an indicator of a relation, (b) using number relations to simplify calculations, and (c) generating, representing, and justifying conjectures about fundamental

properties of number operations. The professional development focused on students' mathematical thinking, with the goal of helping teachers be successful in detailing students' solutions to problems, organizing their knowledge of students' thinking, and using that information to guide instruction (Franke, Carpenter, Levi, & Fennema, 2001; Franke, Fennema, Carpenter, Ansell, & Behrend, 1998). Teachers were encouraged to let students solve problems in their own ways, to engage their students in conversations to help them explicate their thinking, and to debate their reasons for thinking as they did.

Procedures

We videotaped each teacher's class on two occasions within a 1-week period. To capture video and audio for as many students as possible (12 in each classroom), we used two cameras (to record 8 students) and six audio recorders (to record an additional 4 students). Each video camera had two audiofeeds connected to flat microphones (mics; four flat mics in all), so that 8 students could be recorded simultaneously. Each flat mic was positioned between a pair of students. For the students audiotaped only, in addition to the flat mic between 2 students, each student had an individual lapel mic (each attached to a different audiorecorder). Triangulating the recordings from the flat mic and the individual mics helped identify the speaker for students who were audiotaped only.

Classwork Problems

Teachers were asked to engage students in mathematical work around the equal sign and relational thinking on the days that we observed their classes, topics that were central to the professional development program on algebraic thinking. The following are sample problems: (a) $50 + 50 = 25 + \square + 50$ and (b) $11 + 2 = 5 + 8$ (True or False?).

Coding of Student and Teacher Participation

The coding scheme used to analyze these data highlights the questioning practices teachers used to elicit individual student thinking and stimulate mathematical discussion. This scheme to capture student and teacher participation evolved iteratively, growing both out of the literature about mathematical talk and from our review and discussion of the data.

Identifying Segments

Classroom interaction followed a standard structure: The teacher posed a problem, students discussed the solution

within a smaller group composed of two to four students, then the teacher led a discussion of the problem with the entire class before moving on to the next problem. In this article we focus on the whole-class portion of the classroom interaction.

Some of the whole-class discussions for a single problem were quite lengthy, and several students were given a chance to share their thinking. To ensure that we captured the full depth of the discussions and maintained coding in context, we broke the whole-class episodes into segments. We defined a segment as an extended interaction or discussion between the teacher and an individual student, in which that student had at least two conversational turns. Although some of the segments consisted of just two turns, others were longer. The segments began when the teacher called on the student and ended when the teacher either called on another student or moved to a new problem. During segments, teachers sometimes directed the discussion toward the whole class instead of an individual student, and multiple students participated simultaneously without being named by the teacher (we term this interaction *whole-class choral discussion*). If the teacher returned to the original student after a whole-class choral discussion, the segment ended when the interaction with the original student ended. The total number of segments analyzed here is 66. The number of segments per teacher ranged from 10 to 28.

Teacher Questioning

We looked specifically at the questions teachers posed in order to follow up on students' initial explanations and build on student ideas. These often appeared as requests for students to clarify ambiguous explanations, questions directed toward uncovering the reasoning underlying errors students made, requests for further elaboration of problem-solving strategies, and questions to highlight important mathematical ideas.

Our iterative analysis process led us to identify and examine four types of teacher questioning practices that teachers used to help make student thinking explicit: general questions, specific questions, probing sequences of specific questions, and leading questions (these questioning practices are illustrated in later sections). *General questions* were not related to anything specific that a student said. *Specific questions* addressed something specific in a student's explanation. *Probing sequences of specific questions* consisted of a series of more than two related questions about something specific that a student said and included multiple teacher questions and multiple student responses. In *leading questions*, the teacher guided students toward particular answers or explanations and provided opportunities for students to respond.

Table 1
Examples of Student Explanations

Correct and complete	<i>Problem:</i> $20 + 10 = 10 + \square$ Twenty plus 10 is 30, so . . . the equal sign means that you have to be the same, it has to be the same, so if there's a 10 here, then a 20 has to be there. Twenty plus 10 is 30, 10 plus 20 is 30.
Ambiguous or incomplete	<i>Problem:</i> $100 + \square = 100 + 50$ The 50 will go right there because it has to be the same number.
Incorrect	<i>Problem:</i> $4 + 9 = 5 \times 3 - 2$ (True or False?) I thought it was false because $4 + 9$ is 13, and 5×3 is 15. Those two do not match.

In two segments, the teacher asked questions that did not fit into the previous categories; we label these as *other questions*.

It should be noted that teachers engaged in other practices to help make student thinking explicit, such as revoicing or repeating student answers or explanations, describing strategies they thought students used to solve particular problems, and highlighting mathematical ideas in student explanations. We did not analyze these practices because the teacher, rather than the student, was primarily responsible for describing or summarizing student thinking. We focused on teacher questioning because this set of practices was the predominant one used by teachers to help students make their thinking explicit.

Student Participation

Using transcripts of class talk and videotapes, we coded two categories of student participation: (a) accuracy of answer given or implied (correct, incorrect, or none) and (b) nature of explanation given (correct and complete; ambiguous or incomplete; or incorrect; see Table 1 for examples). We also paid attention to the types of explanations that students gave in relation to our algebraic reasoning work (specifically, computational and relational thinking explanations; see Carpenter et al., 2003). Our current analyses consider both kinds of explanations as correct and complete.

Results

Description of Teachers' Questioning Practices

Teachers' directives to students to share their thinking. During whole-class instruction, teachers frequently directed students to share their thinking. In all but one

Table 2
Relationship Between Specific Kinds of Teacher Questioning and Nature of Students' Initial Explanations

Nature of Teacher Questioning of Student Explanation in the Segment	Number of Segments	Initial Student Explanation	
		Correct and Complete	Ambiguous, Incorrect, or Incomplete
Teacher asked question(s) about student explanation	50	18	32
Probing sequence of specific questions	14	5	9
General question	10	3	7
Specific question	18	9	9
Leading question(s)	6	1	5
Other question(s)	2	0	2
Teacher asked no question about student explanation	16	9	7
Total	66	27	39

segment (98%), teachers asked the target students to explain their thinking. In 91% of the segments, the teacher explicitly prompted an explanation by requesting a student explanation at the outset of the segment (73%) or by asking students to explain how they got their answer when they didn't immediately volunteer an explanation (33%). In 76% of segments, the teacher also asked the students to elaborate further on their explanation (these teacher practices are described in detail in the following sections). Teachers also frequently made remarks reminding students to give other students a chance to explain ("Give Rodrigo a chance [to explain]"), to listen to each other's explanations ("Let's listen," "I like the way the people at Table 2 are giving them their full attention," "I like the way Jason is paying close attention to what Marisa and Desiree are about to share"), and to understand each other's thinking ("Let's understand Raven's thinking"). In all segments except two (97%), the target student did provide an explanation (or multiple explanations). Clearly, in these classrooms, students were expected to give explanations about their thinking, and did so.

Teacher questions to prompt further student explaining. In the remaining sections, we analyze the kinds of questions teachers asked to prompt students to clarify or elaborate on their initial explanations. Across the 66 segments, teachers asked questions about student explanations in 50 (76%) of them. These segments contained an assortment of questions, including single general or specific questions, probing sequences of questions, and leading questions, with no one type of question predominating. Additionally, these questions often occurred in combination with each other. For example, of the seven segments that had probing sequences, five also included general questions or single specific questions.

To distinguish segments, we assigned each segment one code in the following order: probing sequences, general

questions, specific questions, leading questions, and other. That is, we coded all segments for probing sequences first, then coded the remaining segments for general questions, and so on. The remaining results use this streamlined coding scheme.

Teacher questioning and students' initial explanations. We next explored whether teachers responded differently depending on the nature of students' initial explanations. To do this, we sorted students' explanations into two categories according to their accuracy and completeness: (a) correct and complete or (b) ambiguous, incomplete, or incorrect. Table 2 shows that whether teachers asked any question about a student explanation did not depend on whether the initial explanation was correct and complete or not, $\chi^2(1, N = 66) = 2.06, p = .15$. Teachers asked about both correct and complete explanations and ambiguous, incomplete, or incorrect explanations. Teachers asked questions about correct explanations in 18 out of 27 cases (67%) and asked questions about ambiguous, incomplete, or incorrect explanations in 32 out of 39 cases (82%).

We also explored whether the nature of teachers' questions depended on whether the initial explanation was correct and complete or not. Table 2 shows that there is no statistical evidence to suggest that whether teachers asked questions about a student's explanation was related to whether the initial explanation was correct and complete or not (Fisher's exact test, $p = .55$).¹ Teachers used nearly all questioning types when students' initial explanations were correct and complete as well as when explanations were ambiguous, incomplete, or incorrect.

Teacher Questioning and Students' Explanations

This section probes the relationship between teachers' questioning practices and students' explanations of their problem-solving strategies. First, we examine whether

Table 3
Relationship Between Teacher Questioning and Students' Elaboration on Their Initial Explanations

Teacher Questioning Category and Initial Student Explanation	Number of Segments	Student Elaborated on Initial Explanation	
		Yes	No
Teacher asked question(s) about student explanation	50	36	14
Correct and complete	18	14	4
Ambiguous, incomplete, incorrect, or no explanation	32	22	10
Teacher asked no question about student explanation	16	0	16
Correct and complete	9	0	9
Ambiguous, incomplete, incorrect, or no explanation	7	0	7
Total	66	36	30

Table 4
Relationship Between Specific Kinds of Teacher Questioning and Students' Elaboration of Their Explanations

Nature of Teacher Questioning of Student Explanation	Number of Segments	Student Elaborated on Initial Explanation	
		Yes	No
Probing sequence of specific questions	14	14	0
General question	10	8	2
Specific question	18	12	6
Leading question(s)	6	2	4
Other question(s)	2	0	2
Total	50	36	14

students elaborated on their explanations during a segment. Second, we consider whether students provided a correct and complete explanation during a segment.

Teacher questioning and student elaboration of explanations. Table 3 gives the number of segments in which students either did or did not elaborate on their explanation according to whether the teacher asked questions about a student's explanation. As can be seen in Table 3, whether students elaborated on their explanations depended greatly on whether teachers asked questions about students' explanations. Not surprisingly, in segments in which teachers did not ask questions in response to student explanations (16 of 50; 32%), students did not provide elaboration. When teachers did ask questions, however, students were much more likely to provide elaboration. Of the 50 segments with teacher follow-up questions, 36 (72%) yielded student elaboration. This difference is statistically significant, $\chi^2(1, N = 66) = 31.65, p < .001$. It should also be noted that for a sizable minority of segments in which the teacher asked follow-up questions (14; 28%), students provided no elaboration of their explanation. This result shows that teacher follow-up questions were not a guarantee of further elaboration by

students of their thinking. A major purpose of the rest of this article is to unpack when follow-up questioning led to further student elaboration and when it did not.

Table 3 also shows that student elaboration was not restricted to incorrect or incomplete explanations. Students elaborated on correct explanations (e.g., providing additional detail) as well as on incorrect, incomplete, or ambiguous explanations. For example, in 14 (39%) of the 36 segments in which students elaborated on their explanation, students had initially given a correct and complete explanation.

Table 4 gives the number of segments in which students elaborated on their initial explanations according to the type of teacher questioning. When teachers asked sequences of specific questions (alone or in conjunction with other questioning types), the target students provided elaboration of their explanations. When the teacher asked general questions or specific questions (but not probing sequences), the target students often provided elaboration of their explanation. Leading questions and other questions did not often lead to student elaboration of their explanations. Differences in student elaboration between questioning types were statistically significant (Fisher's exact test, $p = .002$).

Table 5
Relationship Between Specific Kinds of Teacher Questioning and Accuracy of Students' Explanations

Nature of Teacher Questioning of Student Explanation	Number of Segments	Student Gave Correct Explanation Later in Segment	Student Did Not Give Correct Explanation, but a Correct Explanation Was Given by the Teacher or Another Student	No Correct Explanation Was Given
Probing sequence of specific questions	9	6	1	2
General question	7	1	6	0
Specific question	9	1	6	2
Leading question(s)	5	0	1	4
Other question(s)	2	0	0	2
Total	32	8	14	10

Note: Table includes only segments in which students' initial explanations were not correct and complete.

Teacher questioning and students' success in giving correct and complete explanations. Table 5 gives the number of segments in which students gave correct explanations as a function of teacher questioning type. This table includes only segments in which students' initial explanations were not correct. The target students, whose initial explanations were incorrect, ultimately produced correct and complete explanations primarily in segments with probing sequences of specific questions. Segments with the other questioning types rarely or never yielded correct explanations by the target students (differences between questioning types in whether the target student gave a correct explanation were statistically significant, Fisher's exact test, $p = .001$).

Although only 25% of the segments (8 of 32) had students' providing a correct explanation after they had provided an initial explanation that was not correct, 44% of segments had correct explanations given by others. In only 31% of segments were ambiguous, incorrect, or incomplete explanations left uncorrected.

How Teacher Questioning Played Out During Teacher–Student Interactions

The previous sections have shown that some teacher questioning practices were more likely than others to yield student elaboration of their explanations and to lead to students' giving correct and complete explanations. However, no category of teacher questioning practices was uniformly related to a certain kind of student participation. For example, two thirds of segments with specific questions yielded student elaboration of their explanations, but one third of segments with specific questions did not. In this section, we examine more closely how and why the same kind of teacher questioning was related to such different patterns of student participation across different segments.

Probing sequences of specific questions. Probing sequences of specific questions always elicited further elaboration from the student and usually allowed the student the opportunity to articulate a correct and complete explanation when the initial student explanation was ambiguous, incomplete, or incorrect.

Probing sequences were sometimes used when a teacher was unclear about a student's explanation and was trying to understand the student's thinking that underlay an ambiguous initial explanation. In the following example about commutativity and the equal sign, the teacher did not understand the student's use of the term *partners* (Line 4), so she asked the student a series of questions (Lines 5, 7, and 11) to figure out what the student meant.

- Problem: $200 + 1 = 200 + 1$; $200 + 1 = 1 + 200$; Does it matter which way I put the numbers?
- Ms. Guo: OK, who wants to share out their answers? Who wants to share out? Krystal?
 - Krystal: It doesn't matter which way you put it . . .²
 - Ms. Guo: Oops, OK, hold on. Krystal?
 - Krystal: It doesn't matter the way you put it because it still has a partner.
 - Ms. Guo: Oh! What has a partner?
 - Krystal: The numbers.
 - Ms. Guo: What are you talking about? Could you explain what numbers you are talking about?
 - Krystal: Two hundred and 1, and the 1s.
 - Ms. Guo: One more time.
 - Krystal: Two hundred and 1, and the 1. Two hundred and the 1.
 - Ms. Guo: Two hundred and the 1 like this are partners?
 - Krystal: The 1 and the 1 are partners and the 200 are partners.

By the end of the sequence, it became clear that the student looked across the equal sign to match numbers, which she referred to as partners.

In a few cases, teachers used a probing sequence to uncover a student's incorrect strategy. In the following example, the teacher asked a sequence of questions (Lines 3, 5, 10, and 12) to help the student clarify the strategy he used to arrive at an incorrect answer: He had treated the left side of the number sentence as $10 + 10 + 10$, then subtracted 10 from 30 to obtain 20. He then proceeded to give the answer as 15.

Problem: $10 + 10 - 10 = 5 + \square$

- 1 Ms. Lee: OK, why don't you come and explain how you got the 15? Because other people got 15, too. OK, so if you got 15, let's see if this is the way you got 15.
- 2 Diego: Because there's $10 + 10$. And if you had a plus sign and that would be 30, but there is a take away sign. And then, um, 5 plus, I knew it was 15 because I went, um, I counted—5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20—and then it was 15, and then I put 15.
- 3 Ms. Lee: OK, so if I get you right, I understand you're saying $10 + 10 = 20$. Is that what you're saying? And you're saying 5 plus 15 equals 20? So I have a question for you. What do we do with this number [*points to the last 10*]? What do you do with that?
- 4 Diego: 'Cause if that was a plus sign, that would be 30.
- 5 Ms. Lee: If this was a plus sign . . . If it's a $10 + 10 + 10 = 5 +$ blank, Diego is saying that he would add all this to make what?
- 6 Diego: Thirty.
- 7 Ms. Lee: And then what would that be [*referring to the box*]?
- 8 Diego: That would be, um, 25.
- 9 [*Ms. Lee checks in with the rest of the class to see if they are following. She has them skip count by 10s to see if they arrive at the sum of 30.*]
- 10 Ms. Lee: Diego says $10 + 10 = 20$, and I said what do you do with this? So what did you do with it, Diego?
- 11 Diego: 'Cause it was 10 . . . And then with the plus sign it would be 30, but I take away 10. I took away 10 from the 30 so that equaled 20.
- 12 Ms. Lee: All right, so you're saying you had $10 + 10$, which is 20, and you added another 10?
- 13 Diego: No, I said $10 + 10 . . . 10 + 10$, and that was a plus sign, and that would be 30.
- 14 Ms. Lee: This would be 30, OK.
- 15 Diego: That was 30. Ten + $10 + 10 = 30$.

In this example, the teacher begins using a sequence of probing questions to understand exactly what the student is thinking. The segment continues with the teacher asking more questions about what the student did and highlighting the minus 10 to help him through a correct and complete explanation.

In other instances, teachers used a probing sequence to highlight, clarify, or make explicit a particular part of a student's correct strategy—often for the benefit of other students in the class. In the following example, the teacher used a probing sequence of questions to make explicit the steps used in this student's solution and to highlight the mathematics in it.

Problem: $8 + 8 = 15 + \square$

- 1 Ms. Lee: OK, D'ante, come on up. OK, let's all pay attention because D'ante found a couple ways to solve this. You can take your paper up. That's completely fine. OK. Explain what you did.
- 2 D'ante: I put 16 take away 1 equals 15, and 15 plus . . .
- 3 Ms. Lee: OK, write down what you're saying. Katherine, I want you to watch what he's doing 'cause this will give you another strategy of how to figure out this problem. OK, I have a question for you. Where did you get that 16 from?
- 4 D'ante: I got it from the $8 + 8$.
- 5 Ms. Lee: OK. OK, eyes up here please. He had $8 + 8$ equals 15 plus 1, and he says this 16 is what?
- 6 D'ante: Eight.
- 7 Ms. Lee: Eight. Eight plus 8 . . . so he put that here, and then he moved the unknown to this side, so he did 16 minus blank equals 15. Do you guys see how he did that? Are you allowed to do that? [*She writes on board: $16 - \square = 15.$*]
- 8 Students: Yes.
- 9 Ms. Lee: Are you allowed to move it around like that?
- 10 Students: Yes.
- 11 Ms. Lee: That's OK?
- 12 Students: Yes.
- 13 Ms. Lee: D'ante is saying yes.
- 14 Students: Yes.
- 15 Ms. Lee: OK, where did this 1 come from? What's this?
- 16 D'ante: Fifteen plus 1 equals 16.
- 17 Ms. Lee: So you just . . .
- 18 D'ante: So I just flipped that around.
- 19 Ms. Lee: Can he do that? Can he check his work like that?
- 20 Students: Yes.

The teacher's specific questions (Lines 3 and 15) served to clarify for the class the intermediate steps that the student had not made explicit in his original explanation.

General question in response to student explanation. General questions usually led students to elaborate on their explanation but rarely led them to give a correct and complete explanation if their initial explanation was ambiguous

or incomplete (teachers never responded to an incorrect explanation with only a general question). General questions often signaled that the student should repeat the explanation given. In some cases, the teacher asked the student to repeat the explanation either explicitly (“Can you say it one more time?,” “What did you say?”) or implicitly (“I’m sorry, what did you do right now?”). In other cases, the teacher asked the student to demonstrate on the board the explanation just given (“Could you come up and show us what you mean?”).

Below is an excerpt from a segment in which the teacher asked the class whether the number sentence $1,000 + 200 = 200 + 1,000$ was true and to explain why. The student initially gave a confusing explanation in which she tried to point out the relationship between the numbers on the left side and the right side of the number sentence (Line 1). Her explanation was unclear, in part, because she confused thousands and hundreds. The teacher started to restate the student’s explanation but stopped and asked the student to repeat her original explanation (Line 2). Not only did the student add to her initial explanation by inserting the language “the same answer” (possibly suggesting that she may have thought that the sum of the numbers on the left side of the number sentence was the same “answer” as the sum of the numbers on the right side; Line 3), but she went on to link the numbers to variables A and B (Lines 5 and 7) in an implicit reference to a previous problem, $A + B = B + A$.

Problem: $1,000 + 200 = 200 + 1,000$ (True or False?)

- 1 Lauren: One thousand plus 2,000 equals 200 plus 100 because if 100 has 2,000, then 2,000 needs to have a 1,000.
- 2 Ms. Guo: OK, so you are also doing the partners. I think you mean, if the 1,000, this is 1,000 . . . Can you say it one more time?
- 3 Lauren: One thousand plus 200 are the same answer from 200 plus 1,000 because 100 . . .
- 4 Ms. Guo: One thousand.
- 5 Lauren: One thousand needs to be A and 1,000 is A.
- 6 Ms. Guo: One thousand is A.
- 7 Lauren: And the other 1,000 is A. And 200 is B.

While this student added to her original explanation, her explanation did not become complete. It was not clear whether she understood the role of the arithmetic operation (that is, $1,000 - 200$ is not the same as $200 + 1,000$, even though both sides of the number sentence have the same numbers), nor whether she understood the limits of the partner idea (it became clear that other students believed that number sentences in the form of $A + A = B + B$ were true because both sides had “partners,” that is, numbers that were the same).

Nonetheless, this example shows a common pattern in which a teacher’s general request for the student to explain again was not interpreted literally as a directive to repeat the initial explanation verbatim but was apparently interpreted by the student as a request for further elaboration.

Specific question in response to student explanation. Teachers used specific questions to prompt the students to elaborate a particular aspect of their initial explanations; clarify ambiguous, incomplete, or incorrect parts of explanations; or consider other important elements of the problems. In the majority of cases, a specific question prompted students to elaborate on their initial explanation, although in a substantial minority (33%), the specific question prompted no elaboration. Rarely did a specific question lead students to give a correct and complete explanation if their initial explanation was ambiguous, incomplete, or incorrect.

In the following example, the teacher used a specific question to seek clarification of a student’s ambiguous explanation. To solve the problem $100 + \square = 100 + 50$, the student explained that 50 would go inside the box “because it has to be the same number” (Line 1). The teacher asked a specific question to clarify the ambiguity of the student’s initial explanation (Line 2), leading to a back-and-forth interaction between teacher and student that yielded further clarification of how the student was considering the relationship of the numbers across the equal sign.

Problem: $100 + \square = 100 + 50$

- 1 Eduardo: 100 . . . no, the 50 will go right there because it has to be the same number.
- 2 Ms. Guo: What has to be the same number?
- 3 Eduardo: 150 and the other side has to be 150 too.
- 4 Ms. Guo: OK, so you are adding these together. You said this side has to be 150?
- 5 Eduardo: No.
- 6 Ms. Guo: Oh, what were you saying?
- 7 Eduardo: That they have to be those because . . . ’cause it has to have the same numbers.

The teacher’s question, specific to the student’s initial ambiguous explanation, supported him to clarify his explanation, both for himself and for his classmates. (Although we learn more about Eduardo’s mathematical thinking, he did not indicate whether the “same numbers” had to be on opposite sides of the number sentence or could be on the same side; thus we did not code his explanation as complete.)

Sometimes a teacher’s specific question helped a student make explicit an incorrect explanation. In the following

example, the student references the *Kandra way*, a strategy previously identified in the class, in which students considered pairs of corresponding numbers in problems of the type $A + B = B + A$. The teacher's specific question (Line 3) helped this student make explicit her misunderstanding that a pair of numbers in one part of a number sentence required other numbers to be paired with identical elements, regardless of the operation in the number sentence (Line 4).

Problem: $50 + 500 - 500 = \square$

- 1 T: You did it a different way, Lizeth? Can you explain what you did here?
- 2 Lizeth: [*draws line connecting the 500 and 500 and the 50 and the box*] I did the Kandra way.
- 3 T: The Kandra way? OK, and what's the Kandra way? Can you explain to me what those lines mean?
- 4 Lizeth: The lines mean that there are two 500s so I know that 50 is over here and . . . you are supposed to be 50.
- 5 T: So you see that there's two 500s right here and two 50s right here, so it has to be the same? Does it matter that this is a minus sign? Because usually when we do this with the Kandra way, we usually have a 50 plus 500 and then the equal sign. [*writes: $50 + 500 = 500 + 50$*] When she made it the same . . . Does it matter that this is a minus sign to you? Does that make a difference, do you think? It does make a difference? OK, I'm going to come back to this.

The teacher's specific question supported this student to make explicit her misappropriation of the Kandra way, and the teacher used this opportunity to continue the discussion of this strategy with the class.

Finally, teachers asked a specific question not only when student explanations were incomplete, ambiguous, or incorrect, but also when they were initially correct and complete. In this example, the student explains that he added the 11 and 2 to get 13 (using a "caret" notation to indicate this sum). He then explains that he subtracts 10 from the 13. The teacher asks a specific question about the 10 (Line 4). Her question attempts to highlight the role of the 10 in this problem.

Problem: $11 + 2 = 10 + \square$

- 1 Andrew: I put 11 here, put a 2 right here, then I plussed it, and it was 13. I put take away . . . (*inaudible*) take away [*holds fingers up*]. I wrote 13 right here. I put right here a caret and put 13. I put here a 13. Three, 13 take away, take away 10 and then I minus, I minus [*counting on fingers*] . . . 10. And then (*inaudible*).
- 2 Ms. Lee: Wow. OK. This is really interesting. OK, let's look at this. Does everybody understand how you got 11 plus 2 equals 13?

3 Students: Yeah.

4 Ms. Lee: OK. Why did you minus 10? And where did you get that 10 from?

5 Andrew: 'Cause on the 3 I added first . . . I went 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 (*inaudible*).

6 Ms. Lee: OK, is there anywhere else in the problem that has a 10?

7 Andrew: Yes, here [*points to the 10 to the right of the equal sign*].

8 Ms. Lee: There. OK, does anyone see that connection?

While the student initially explained a strategy that works, computing one side, the teacher continued to ask questions, providing an opportunity for the class to more fully understand the student's solution as well as the problem.

Leading questions in response to student explanation.

When using leading questions, teachers often asked questions that included aspects of the problem that they wanted the students to focus on. The teacher encouraged students to give more detail, but it was the teacher who was primarily responsible for getting the explanation out on the table. In this example, the student completed what the teacher was leading her to say, resulting in a more detailed explanation that connected both sides of the number sentence.

Problem: $100 = 50 + \square$

1 Lauren: I think 50 goes inside the box because $50 + 50 = 100$.

2 Ms. Guo: Fifty + 50 = 100 and so 100 is the same as . . .

3 Lauren: One hundred.

In this particular example, the teacher draws on the student's explanation, but this is not always the case with leading questions. Although leading questions are usually connected to student explanations, they often do not draw out details of the student's initial explanation or build on the mathematical work the student presented.

Summary

The segments examined here show that teachers consistently asked students to explain their thinking. As teachers posed the problem, they showed that they expected students to share their thinking. And, following students' solution of the problem, teachers asked students to explain. This was similar across all segments. The follow-up to these initial questions, however, varied across segments. Sometimes teachers followed up a student's explanation with a general question; other times

the teachers asked questions or sequences of questions specific to what students said. The segments also included bundles of questions, leading questions, and leading sequences of questions. What teachers did to make student thinking explicit, then, varied.

Students' responses also varied, sometimes according to the kinds of follow-up questions teachers asked. Many times these questions supported students to make their thinking more explicit and even helped them add to it in ways that led to a correct and complete strategy being shared. However, there were also instances of follow-up questions, of every type, that did not lead to students' sharing complete strategies.

Discussion

This study provides evidence about how teachers' questions can support students to be more explicit and detailed in their explanations (Sfard & Kieran, 2001). The literature is clear that supporting students' explanations requires teachers to not only provide sufficient time and appropriate tasks but also press for justification and explanation (Kazemi & Stipek, 2001; Silver, 1996; Silver & Smith, 1996). In the classrooms examined here, the teachers frequently followed up on students' initial responses, and they did so in a variety of different ways. At times they probed one student in a focused manner over a series of turns. At other times they asked one specific question related to something the student had said, or they asked a general question to prompt the student give additional explanation. Sometimes they asked leading questions, and sometimes they did none of these.

Follow-up questions, however, did not guarantee further student explanation. We found that the particular moves teachers made after their initial question seeking an explanation mattered for students' opportunities to make their explanations explicit. Asking a probing sequence of specific questions, asking a single specific question, and asking a general question often resulted in students' elaborating on their initial explanation, but only the first teacher questioning practice—asking a probing sequence of specific questions—frequently helped students provide a correct and complete explanation after they initially provided an explanation that was not correct and complete. Most important, uncovering details of students' strategies often required multiple specific questions, each one focused on an element of a student's explanation. In this way, the teachers focused on what students said in relation to the critical mathematical ideas and pressed students to make their thinking explicit (similar to the "focusing" question patterns described by Wood, 1998). When students' initial explanations were

incomplete or ambiguous, the teacher's multiple, more focused questions helped students to make sense of their ideas in relation to the mathematics and thus provide complete and correct explanations. When students' initial explanations were incorrect, the teachers helped students correct the explanations by asking questions that showed that the teacher understood students' mathematical thinking and helped the students identify errors or clarify misunderstandings. In both cases the teachers' multiple, focused questions connected student ideas to the mathematics being worked on.

These probing sequences of specific questions had potential benefits for all participants in the class. First, they enabled the teacher to more fully understand student thinking and, therefore, to make more informed instructional decisions, such as what additional problems to pose and what questions to ask of other students to reveal the nature of their understanding. Second, they helped the student being questioned clarify, solidify, and correct his or her own thinking. Third, they gave opportunities for other students in the class to connect their own thinking to what was being said, potentially enabling them to correct their own misunderstandings.

Single questions, whether specific or general, in contrast, were not always sufficient to uncover enough details of students' strategies to make it known what students were thinking. When ambiguous descriptions of strategies were left on the table, the teacher had to make assumptions about the strategy being used, and as our examples demonstrated, these assumptions were not always correct. Incorrect assumptions could lead the teacher to make unhelpful subsequent instructional decisions, such as deciding that posing further problems or questioning other students was unnecessary. Furthermore, misunderstandings left unaddressed helped neither the student being questioned nor other students in the class learn how to resolve them.

Teachers also asked leading questions. These questions are related to Wood's (1998) idea of funneling. Here the teacher assumed much of the mathematical work while supporting students when moving them through correct and complete explanations. Unlike probing questions, leading questions did not always relate to students' mathematical thinking but instead corresponded to strategies the teacher thought would enable students to solve the problem. We do not argue here that leading questions prevent students from giving correct and complete explanations. Rather we suggest that further research is needed to understand the role of leading questions as our analyses suggest that leading questions did not provide opportunities for students to build on their own understanding. Further research should clarify the contexts in

which leading questions occur and how these questions influence student participation and learning.

This study shows that teachers' questions can position the student thinking in relation to the mathematics in ways that support student understanding. Our analyses provide evidence that much questioning occurs after teachers' initial questions asking students to explain how they solved problems. Further research should investigate how to help teachers consider how their follow-up questions do or do not help students further their mathematical understanding.

Finally, this study showed that learning the details about teachers' questioning practices and students' responses required hearing what students said in relation to teacher questions, which has methodological implications for future research. It is imperative not only to be able to hear the details of what many students say, but also to examine student participation in relation to teacher participation and the context of the classroom. This type of analysis is difficult, as one cannot strip what teachers say from the context in which it happens or from how students engage with classroom interaction. Yet this close attention to what students say and do in relation to what a teacher says, in conjunction with a variety of student outcomes, can help us understand the ways in which teachers can support students' mathematical understanding through classroom dialogue that supports students in explaining their thinking.

Notes

1. For all contingency table analyses with small expected cell counts (less than 5), we used a Fisher's exact test (Fisher, 1935).

2. In all examples of classroom dialogue, the ". . ." indicates a trailing-off tone of voice rather than dialogue that has been omitted. Any omission of dialogue is explicitly noted.

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